Introduction

In this section, the lessons focus on estimating, measuring, and classifying angles, identifying congruence and noncongruence, investigating complimentary, supplementary, and vertical angles, and constructing angle bisectors.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and practice the skills introduced in the ARI Curriculum Companion.

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

For the lessons in this section, you will need the materials listed at right.

MATERIALS SUMMARY
3 x 5 inch index cards
Centimeter rulers
Colored paper
Colored pencils
Compasses
Craft sticks
Patty paper, tracing paper, or wax paper
Permanent markers
Protractors

Standards of Learning

The following Standards of Learning are addressed in this section:

5.13   The student will measure and draw right, acute, and obtuse angles and triangles, using appropriate tools.

5.14   The student will classify angles and triangles as right, acute, or obtuse.

6.13   The student will
a)   estimate angle measures, using 45°, 90°, and 180° as referents, and use the appropriate tools to measure the given angles; and
b)   measure and draw right, acute, and obtuse angles and triangles.

6.15   The student will determine congruence of segments, angles, and polygons by direct comparison, given their attributes. Examples of noncongruent and congruent figures will be included.

6.16   The student will construct the perpendicular bisector of a line segment and an angle bisector.

8.6   The student will verify by measuring and describe the relationships among vertical angles, supplementary angles, and complementary angles and will measure and draw angles of less than 360°.

Table of Contents

The following lessons are included in this section. Click (or CTRL+click) on each to jump to that lesson.

✱ SOL 5.14

Prerequisite SOL

4.14, 4.15a

Lesson Summary

Students define the term *angle* and classify a given angle as right, obtuse or acute. (45 minutes)

Materials

"What I Know About _____" worksheets	Craft sticks	"Station Answers" worksheets
"Index Card Angles" worksheets	Colored paper	"Reflection" worksheets
3 x 5 inch index cards		

Vocabulary

angle. Two rays with a common endpoint.
right angle. An angle that measures exactly 90°.
acute angle. An angle that measures more than 0° but less than 90°.
obtuse angle. An angle that measures more than 90° but less than 180°.
vertex. The common endpoint of the rays forming an angle.
ray. Part of a line starting at a particular point and extending indefinitely in one direction.

Warm-up

Distribute the "What I Know About _____" worksheets. Write the word *angle* on the board, and have students fill in the blank at the top of the page with the word *angles.* Give students three to five minutes to write down everything they know or think they know about angles. When time is up, put students in pairs to share and expand their personal lists. Have the pairs share their ideas with the whole class, and discuss what the students know about angles. Record their responses on the board for everyone to see and add to their lists.

Lesson

1. Distribute an "Index Card Angles" worksheet, an index card, and two craft sticks to every student.

2. Explain to students that there are three types of angles and that they are going to create them, using the craft sticks and index cards.

3. Have students place their two craft sticks along the outside of the corners of the index card. This forms a right angle. Students may refer to the "Index Card Angles" worksheet for assistance. To help students remember what a **right angle** looks like, tell them the story of Rachel Right. In class, Rachel Right always has the right answer so she always raises her hand to answer the questions. Have students raise their hand to show the vertical ray of the right angle.

4. Have students keep the horizontal craft stick in place and slant the vertical craft stick to the right. Students may use the "Index Card Angles" worksheet for assistance. Ask students if this angle is larger or smaller than the right angle. (smaller) Tell students the story of Annette Acute. She was always worried about her appearance. She was so **cute** that she always had her hand in her hair making sure that she looked just perfect. Have students pretend to primp: this forms an **acute angle** with the elbow as the vertex.

5. Have students keep the horizontal craft stick in place and slant the vertical craft stick far to the left so that the angle takes up the index card and much more. Students may use the "Index Card Angles" worksheet for assistance. Ask if this angle is larger or smaller than the right angle. (larger) Tell students the story of Oliver Obtuse. He could never keep his hands to himself. He was always busy bothering the student beside him. He would always reach over to tap that student on the shoulder. Have students reach out as if they were going to tap the person beside them on the shoulder: this forms an **obtuse angle** with the elbow as the vertex.

6. Before class, print each "Station" sheet on a different colored sheet of paper, and place each sheet at a certain spot ("station") in the room. Give a "Station Angles" worksheet to each student. Have students take turns going to each station and classifying each angle as right, acute, or obtuse. Show them how to do this by placing the corner of their index card on the vertex of the angle and comparing whether the angle is smaller than, larger than, or exactly the same as a right angle.
7. Check answers as each student completes the task.

Reflection

Have students complete the "Reflection" worksheet.

Name: _____

What I Know About _____

Here's what I know about _____:

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____

Name: _____
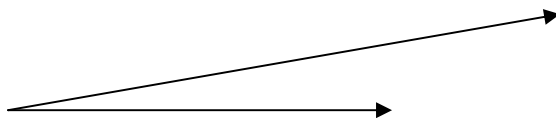
Index Card Angles
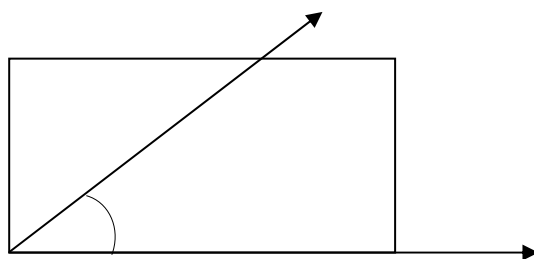
acute angle
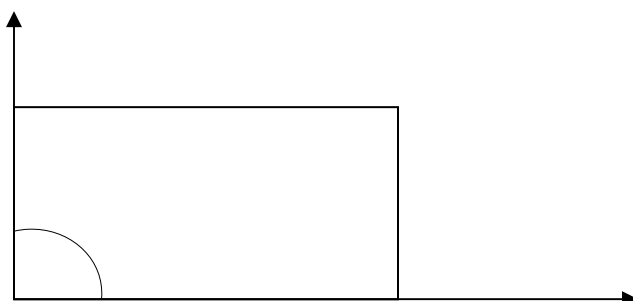
right angle
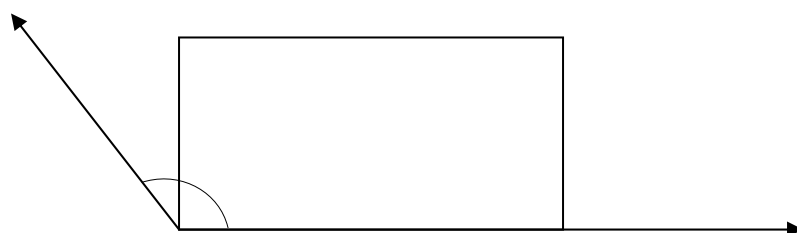
obtuse angle

Station 1
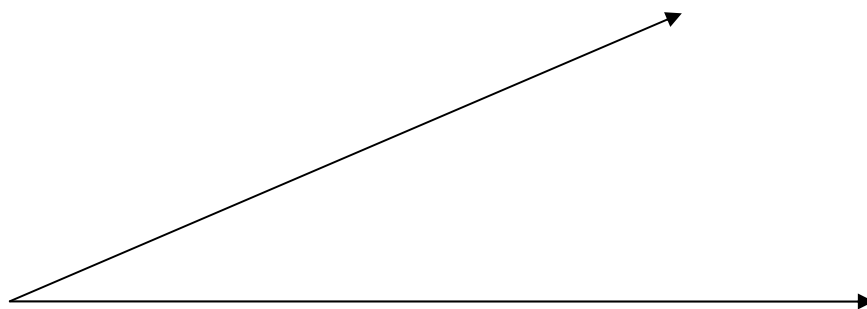
Station 2
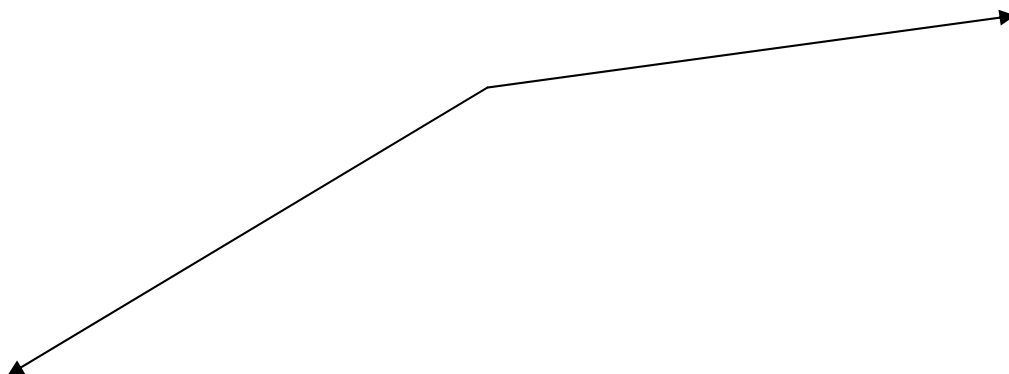
Station 3

Station 4

Name: _____

Station Angles

At each station, examine the angle, and decide if it is an acute, obtuse, or right angle. Circle the correct answer.

Station 1
A   acute
B   obtuse
C   right

Station 2
A   acute
B   obtuse
C   right

Station 3
A   acute
B   obtuse
C   right

Station 4
A   acute
B   obtuse
C   right

Name: <u>ANSWER KEY</u>

Station Angles

At each station, examine the angle, and decide if it is an acute, obtuse, or right angle. Circle the correct answer.

Station 1
(A)   acute
B   obtuse
C   right

Station 2
A   acute
(B)   obtuse
C   right

Station 3
(A)   acute
B   obtuse
C   right

Station 4
A   acute
B   obtuse
(C)   right

Name: _____

Reflection

1. To the right is a practice SOL question. Circle your answer.

2. Explain why you chose that answer.

    _____

    _____

3. Give another example of a time at which the hour and minute hands of the clock form this type of angle.

    _____

    _____

4. Look around the room for angles. List four examples below.

    1. _____

    2. _____

    3. _____

    4. _____

What type of angle is formed between the hands of the clock shown below?
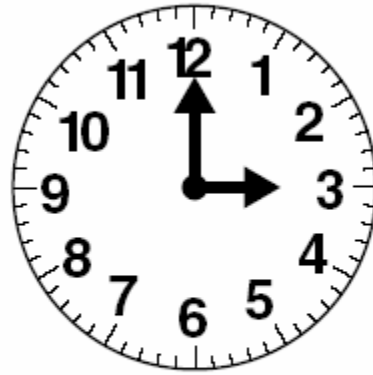
A   Right
B   Acute
C   Obtuse
D   Straight

Name: <u>**ANSWER KEY**</u>

Reflection

1. To the right is a practice SOL question. Circle your answer.

   <u>A</u>

2. Explain why you chose that answer.

   <u>Because both hands of the clock meet</u>
   <u>together to form what looks like a corner.</u>

2. Give another example of a time at which the hour and minute hands of the clock form this type of angle.

   <u>Some possible answers: 3:30, 9:00, 9:30</u>

3. Look around the room for angles. List four examples below.

   <u>Answers will vary, e.g., "corners of the wall,</u>
   <u>edges of furniture, posters, pencils, floor tiles."</u>

What type of angle is formed between the hands of the clock shown below?
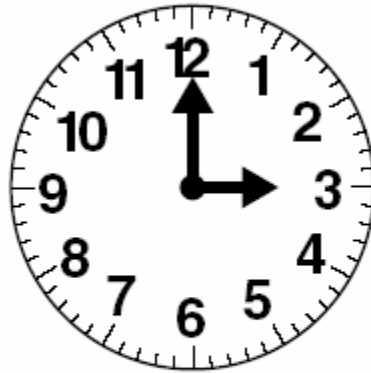
A Right
B Acute
C Obtuse
D Straight

✱ SOL 5.13, 5.14, 6.13b

Prerequisite SOL

4.14, 4.15

Lesson Summary

Students name, measure, and classify angles. (40 minutes)

Materials

"Warm-up" worksheets   Protractors
"Naming and Measuring Angles" worksheets   "Reflection" worksheets
"Types of Angles Vocabulary" handouts

Vocabulary

angle. Two rays with a common endpoint.
right angle. An angle that measures exactly 90°.
acute angle. An angle that measures more than 0° but less than 90°.
obtuse angle. An angle that measures more than 90° but less than 180°.
vertex. The common endpoint of the rays forming an angle.
ray. Part of a line starting at a particular point and extending indefinitely in one direction.
protractor. A tool used to measure angles in degrees.

Warm-up

Distribute the "Warm-up" worksheets, and have students classify each angle by writing the correct term in the space provided. After students have completed the worksheet, go over the answers.

Lesson

1. Distribute copies of the "Naming and Measuring Angles" worksheet and the "Types of Angles Vocabulary" handout.

2. Have students compare the angles from the warm-up worksheet to the angles on the "Naming and Measuring Angles" worksheet.

3. Demonstrate the two methods of naming an angle — by its vertex only, or by three points on the angle with the vertex listed between the other two points. Then, ask the students to name each angle on the worksheet three ways (e.g., ∠PBK, ∠B, ∠KBP). (Teacher Note: Students often confuse "naming" and "classifying" an angle. An effective way for them to remember the difference between the two is for them to equate this to their initials, which are most often comprised of three letters — the first letter of their first, middle, and last *names*. We *name* angles in the same manner — with letters. We *classify* angles with the words — *acute, obtuse,* and *right.*)

4. Pass out protractors, and teach students how to use the protractor if they are not familiar with the tool. Place the protractor on the angle so that the center is on the vertex and one ray goes through 0° on the protractor. The point where the other ray of the angle meets the protractor is the measure of the angle. It is normal to have a difference of two to three degrees over or under. Once students know how to measure, have them measure each angle on the worksheet. (Teacher Note: Many students have difficulty using a protractor. They have problems knowing what numbers to use — top or bottom. Model for students the strategy of first asking, "Is this angle acute or obtuse?" before they measure. This will help them know which numbers to use. Also remind students that if the rays of the angle are short they may use a straightedge to extend them to reach the protractor measurements.)

5. After an exact measure is given, the student can use the "Types of Angles Vocabulary" handout, if necessary, to classify the angle as acute, obtuse, or right, and record the answers on the line.
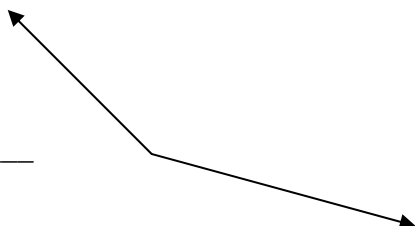
Reflection

Have students complete the "Reflection" worksheet.
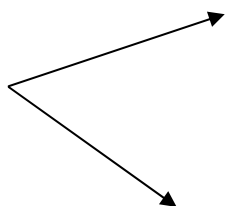
Name: _____

Warm-up

Classify each of the following angles as acute, obtuse, or right.
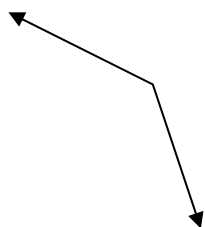
1. _____

2. _____

3. _____

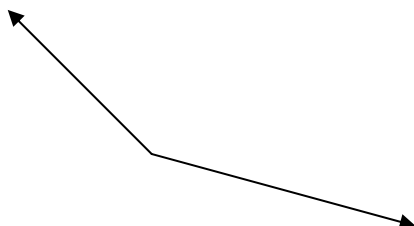4. _____

Name: <u>ANSWER KEY</u>
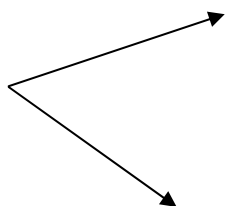
Warm-up

Classify each of the following angles as acute, obtuse, or right.

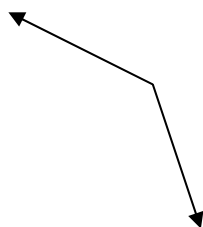1. <u>obtuse</u>

2. <u>acute</u>

3. <u>right</u>

4. <u>obtuse</u>

Types of Angles Vocabulary

acute angle

An angle that measures more than 0° but less than 90°.

right angle

An angle that measures exactly 90°.

obtuse angle

An angle that measures more than 90° but less than 180°.

Name: _____

Naming, Measuring, and Classifying Angles

Name, measure, and classify the following angles.

1. Name: _____

    Measure: _____
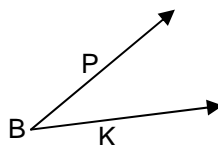
    Classification: _____

2. Name: _____

    Measure: _____

    Classification: _____
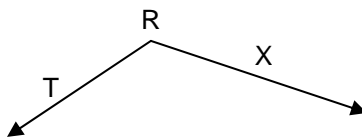
3. Name: _____

    Measure: _____

    Classification: _____
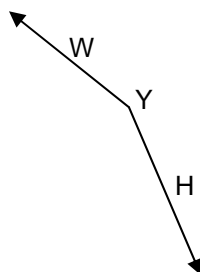
4. Name: _____

    Measure: _____

    Classification: _____
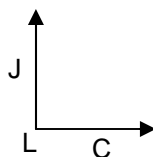
Name: ANSWER KEY

Naming, Measuring, and Classifying Angles

Name, measure, and classify the following angles.

1. Name: ∠PBK, ∠B, ∠KBP

    Measure: 34°

    Classification: acute
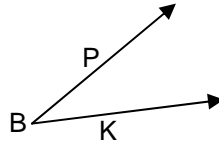
2. Name: ∠TRX, ∠R, ∠XRT

    Measure: 127°

    Classification: obtuse
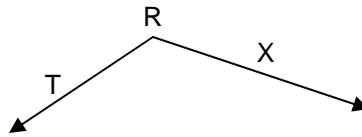
3. Name: ∠WYH, ∠Y, ∠HYW

    Measure: 150°

    Classification: obtuse
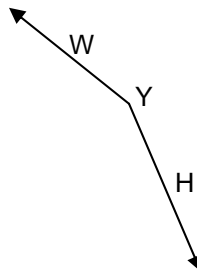
4. Name: ∠JLC, ∠L, ∠CLJ

    Measure: 90°

    Classification: right
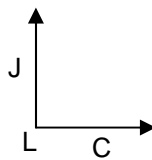
Name: _____

Reflection

1. To the right is a practice SOL question. Circle your answer.

2. Explain why you chose that answer.

_____

_____

_____

_____

3. Name Laura's angle three ways.

_____

4. Classify Laura's angle.

_____

5. Using a straightedge, draw an angle of your own at the bottom of this page. Label the vertex with a point and letter and include two other points. Name your angle three ways. Use a protractor to measure your angle. Classify your angle.

_____

_____

_____

_____

Laura connected points Y and Z to make one side of an angle. Which other point should she connect to point Y in order to make a 28° angle?
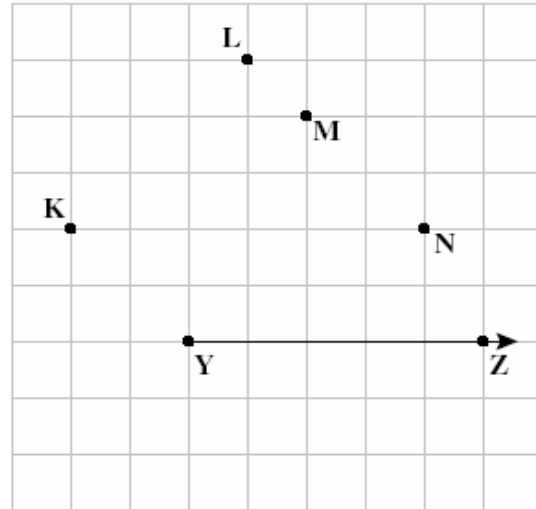
A K
B L
C M
D N

Name: <u>**ANSWER KEY**</u>

Reflection

1. To the right is a practice SOL question. Circle your answer.

    <u>D</u>

    Laura connected points Y and Z to make one side of an angle. Which other point should she connect to point Y in order to make a 28° angle?

2. Explain why you chose that answer.

    <u>I put the center of the protractor on point Y and lined ray YZ on the 0° line. I looked at the mark for the protractor for 28°. Point N was on that mark, which is answer D.</u>

3. Name Laura's angle three ways.

    <u>∠NYZ, ∠Y, ∠ZYN</u>

4. Classify Laura's angle.

    <u>acute</u>

5. Using a straightedge, draw an angle of your own at the bottom of this page. Label the vertex with a point and letter and include two other points. Name your angle three ways. Use a protractor to measure your angle. Classify your angle.

    <u>Answers will vary.</u>
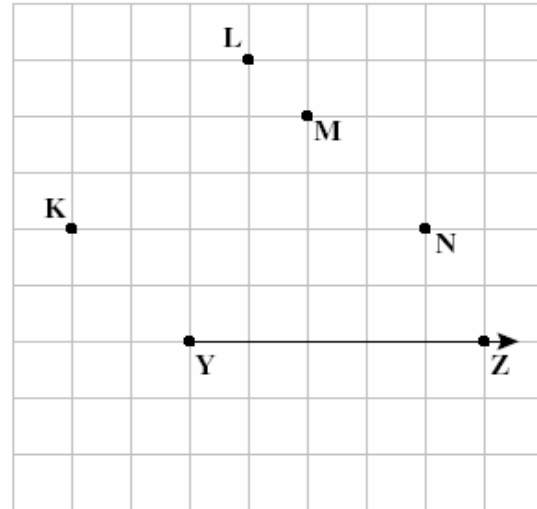
A K
B L
C M
Ⓓ N

✶ SOL 6.13a

Prerequisite SOL

5.13, 5.14

Lesson Summary

Students use a guideline protractor in intervals of 45° to estimate the measure of angles before finding the exact measures. (45 minutes)

Materials

Wax paper, patty paper, or tracing paper	"Warm-up" worksheets
Permanent markers	"Estimating and Measuring Angles" worksheets
Protractors	"Reflection" worksheets

Vocabulary

angle. Two rays with a common endpoint.
right angle. An angle that measures exactly 90°.
acute angle. An angle that measures more than 0° but less than 90°.
obtuse angle. An angle that measures more than 90° but less than 180°.
vertex. The common endpoint of the rays forming an angle.
ray. A part of a line starting at a particular point and extending infinitely in one direction.

Warm-up

Distribute the "Warm-up" worksheets, and have the students answer the questions. Go over the answers before going on to the lesson.

Lesson

1. Distribute a piece of wax paper (or tracing paper or patty paper), a protractor, and a permanent marker to each student.
2. Have students draw a straight line on the wax paper with their permanent marker. The line should be 5 to 6 inches in length.
3. Direct the students to place their protractor so that both ends of the line are on the 0° marks, and have them mark a point at the center mark, the 45° mark, the 90° mark, and the 135° mark.
4. Have students connect each point to the center point with a straight line. (See drawing on next page.) This creates a "guideline protractor" marked in intervals of 45° — a great tool for estimating the measurement of angles.
5. Direct students to label the guideline protractor with the inside and outside measurements so that it looks like a regular protractor.
6. Explain to students how they can use their guideline protractor just like a regular protractor as a tool to estimate angles. Students place the guideline protractor on top of the angle they want to measure, making sure that the center is on the vertex and one ray is on the 0° mark of the protractor. They then estimate the measurement of the angle by seeing between which two measurements the angle lies.
7. Distribute copies of the "Estimating and Measuring Angles" worksheet. Have students use their guideline protractor to estimate the measurement of each angle and to record their estimates. Remind students to extend the rays, if needed, and to decide whether the angle is acute or obtuse before measuring.
8. After all angle measurement have been estimated, have students use a regular protractor to find the actual measurement of each angle and record these actual measurements.
9. Ask the students to compare their estimated measurements to their actual ones. Were they close? Have a discussion about how close an estimate needs to be for it to be useful. In what situations is an estimate useful?

Reflection

Have students complete the questions on the "Reflection" worksheet.

Guideline Protractor

90°

45°
135°

135°
45°

0°
180°

180°
0°
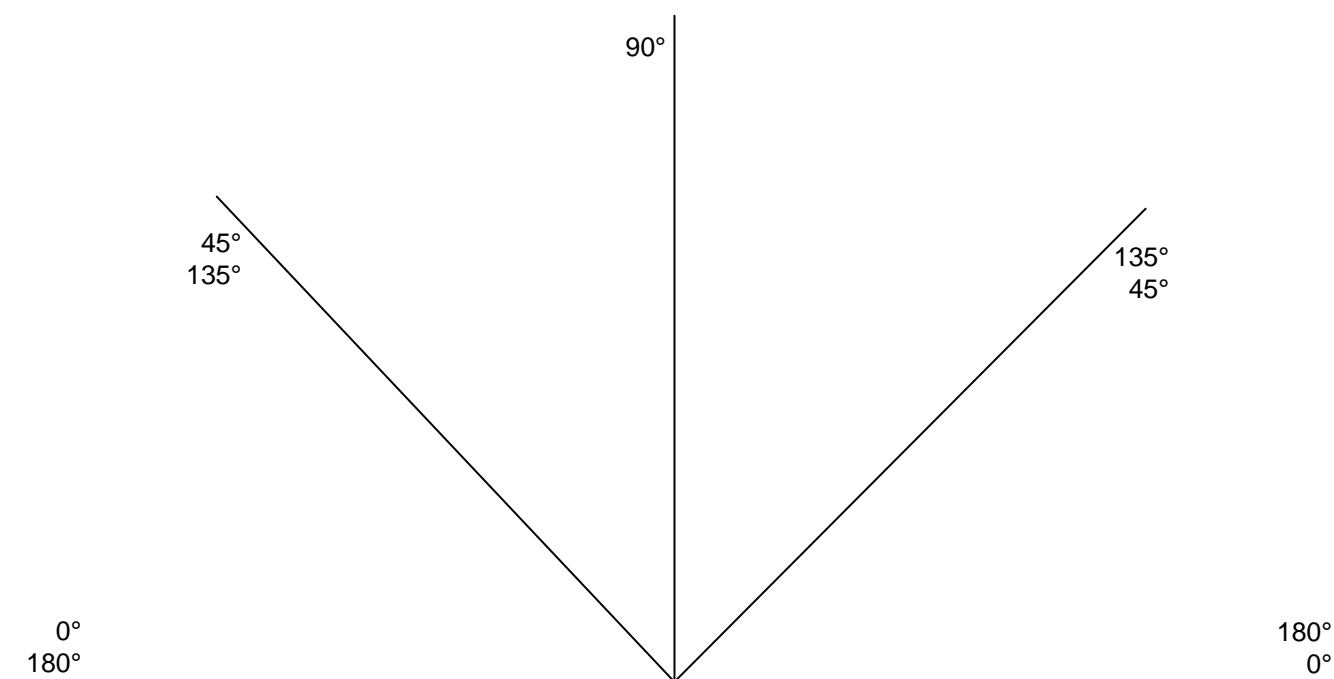
Name: _____
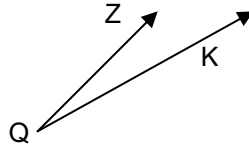
Warm-up

Name, measure, and classify the following angles.

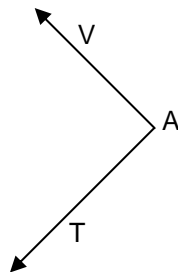1. Name: _____

    Measure: _____

    Classification: _____

    Z

    K

    Q
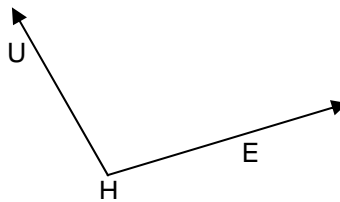
2. Name: _____

    Measure: _____

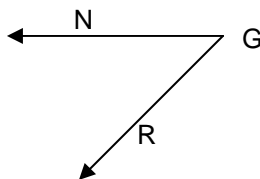   Classification: _____

    V

    A

    T

3. Name: _____

    Measure: _____

    Classification: _____

    U

    E

    H

4. Name: _____

    Measure: _____

    Classification: _____

    N

    G

    R

5. What does the word *estimation* mean to you?

    _____

    _____

    _____

    _____

Name: <u>ANSWER KEY</u>

Warm-up

Name, measure, and classify the following angles.

1.   Name: <u>∠ZQK, ∠Q, ∠KQZ</u>

     Measure: <u>12°</u>

     Classification: <u>acute</u>
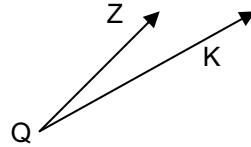
2.   Name: <u>∠VAT, ∠A, ∠TAV</u>

     Measure: <u>90°</u>

     Classification: <u>right</u>
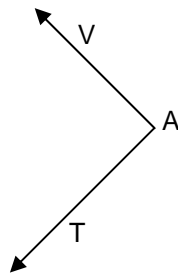
3.   Name: <u>∠UHE, ∠H, ∠EHU</u>

     Measure: <u>101°</u>

     Classification: <u>obtuse</u>
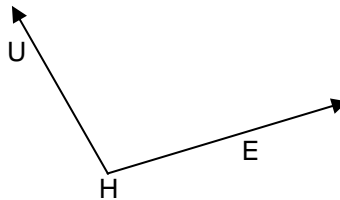
4.   Name: <u>∠NGR, ∠G, ∠RGN</u>

     Measure: <u>45°</u>

     Classification: <u>acute</u>
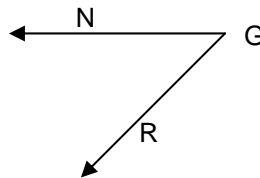
5.   What does the word *estimation* mean to you?

     <u>Some possible responses: "A guess at the answer." "Trying to get close to the real</u>
     <u>answer." "An approximation."</u>

Name: _____

Estimating and Measuring Angles

Use your guideline protractor to estimate the measurement of each angle. Use your regular protractor to get the exact measurement.

1. Estimation: _____

   Actual: _____
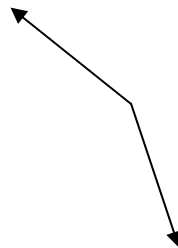
2. Estimation: _____

   Actual: _____
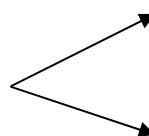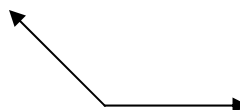
3. Estimation: _____

   Actual: _____
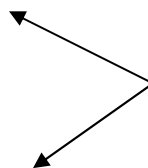
4. Estimation: _____

   Actual: _____
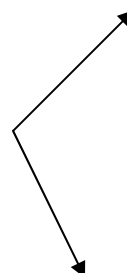
5. Estimation: _____
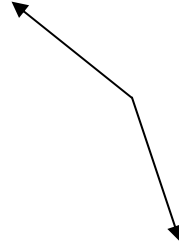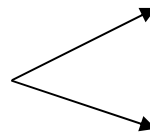
   Actual: _____

6. Estimation: _____

   Actual: _____
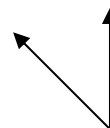
Name: <u>ANSWER KEY</u>

Estimating and Measuring Angles

Use your guideline protractor to estimate the measurement of each angle. Use your regular protractor to get the exact measurement.

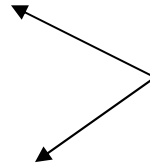1. Estimation: <u>between 135° and 180°, or about 150°</u>

    Actual: <u>148°</u>

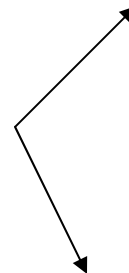2. Estimation: <u>between 45° and 90°, or about 47°</u>

    Actual: <u>46°</u>

3. Estimation: <u>between 45° and 90°, or about 68°</u>

    Actual: <u>62°</u>
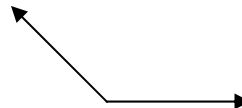
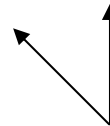4. Estimation: <u>between 90° and 135°, or about 115°</u>

    Actual: <u>110°</u>

5. Estimation: <u>exactly on 135°</u>

    Actual: <u>135°</u>

6. Estimation: <u>exactly on 45°</u>

    Actual: <u>45°</u>

Name: _____

Reflection

1. To the right is a practice SOL question. Use your guideline protractor only to find the correct answer. Circle your answer.

2. Explain how you used your guideline protractor to find the correct answer.
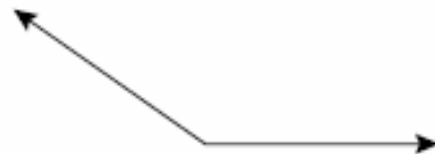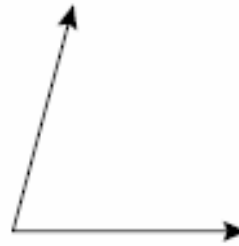
   _____

   _____

   _____

   _____

   _____

   _____

   _____

3. How could you figure out the answer without a protractor or a guideline protractor?

   _____

   _____

   _____

   _____

   _____

Which of the following angles measures closest to 170°?

A

B

C

D

Name: <u>**ANSWER KEY**</u>

Reflection

1. To the right is a practice SOL question. Use your guideline protractor only to find the correct answer. Circle your answer.

    <u>D</u>

2. Explain how you used your guideline protractor to find the correct answer.

    <u>I placed the center of the guideline protractor on the vertex of the angle. I made sure that one ray was on the 0° mark. I looked at the other ray for the measurement. The angle was between 135° and 180°. It was closest to 180°. Therefore, that was the correct answer.</u>

3. How could you figure out the answer without a protractor or a guideline protractor?

    <u>I know that a straight line is 180°. 170° is only 10° away from a straight line. The angle is answer choice D because it is almost a straight line.</u>

Which of the following angles measures closest to 170°?

A

B

C

D
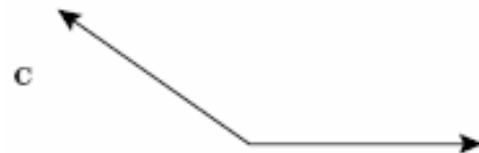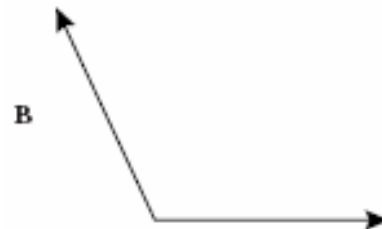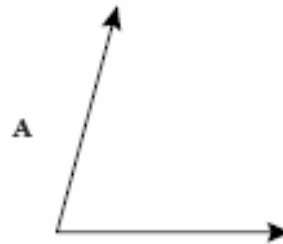
✱ SOL 6.15

Prerequisite SOL

5.13, 5.14, 6.13

Lesson Summary

Students compare line segments, angles, and polygons for congruence, using tracing and direct measurement. (40 minutes)

Materials

Protractors	"Warm-up" worksheets
Centimeter rulers	"Determining Congruence" worksheets
Permanent markers	"Determining Congruence Table" worksheets
Patty paper or tracing paper	"Reflection" worksheets

Vocabulary

line segment. Two points, called the end points of the segment, and all the points between them; part of a line.

angle. Two rays with a common endpoint.

ray. Part of a line starting at a particular point and extending infinitely in one direction.

congruent. Having the same size and shape.

Warm-up

Distribute the "Warm-up" worksheets, and if necessary, explain to students or remind them how to measure using a centimeter ruler. After they have completed the worksheet, go over the answers before going on to the lesson.

Lesson

1. Distribute the "Determining Congruence" and "Determining Congruence Table" worksheets.

2. Part A: Have students measure each pair of line segments, using a centimeter ruler, and record their measurements on the "Determining Congruence Table." Have them compare the two measurements of the pairs of segments. If the two measurements are the same, the two line segments are congruent; if the two measurements are different, the two line segments are noncongruent. (Teacher Note: It is important for students to fully understand that line segments and other shapes may be congruent even if they look different because they are oriented differently. If students need reinforcement with this concept, have them practice reorienting pairs of congruent line segments and pairs of congruent figures.

3. Part B: Have students measure each angle in degrees, using a protractor, and record the measures of the angles in the "Determining Congruence Table." Have students compare the measures of the two angles: if the two angles have the same measure, then the two angles are congruent; if the two angles have different measures, then the two angles are not congruent.

4. Part C: Distribute patty paper or tracing paper and permanent markers. Have students trace on the paper one of the polygons in each pair, using a permanent marker, and compare the pair of polygons by placing the traced polygon on top of the other polygon in the pair. If they are an exact match in size and shape, then the two polygons are congruent; if the two polygons differ in size and/or shape, then the two polygons are noncongruent.

Reflection

Have students answer the questions on the "Reflection" sheet. Review the responses as you walk around and check the students' work.

Name: _____

Warm-up

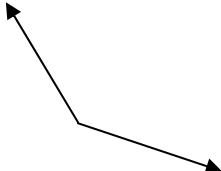Measure each line segment, using a centimeter ruler, and record each measurement.
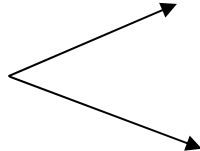
1. _____   Measure = _____
   K   M

2. _____   Measure = _____
   G   T

3. _____   Measure = _____
   W   R

4. _____   Measure = _____
   E   L

Measure each angle in degrees, using a protractor, and record each measure.

5.   Measure = _____
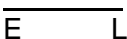
6.   Measure = _____

7.   Measure = _____

8.   Define *congruent*: _____
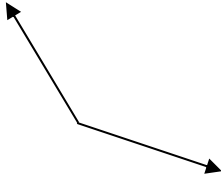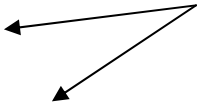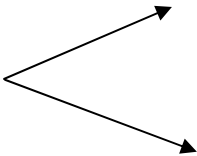
Name: <u>ANSWER KEY</u>

Warm-up

Measure each line segment, using a centimeter ruler, and record each measurement.

1.
K   M

Measure = <u>4 1/2 cm or 4.5 cm</u>

2.
G   T

Measure = <u>7 9/10 cm or 7.9 cm</u>

3.
W   R

Measure = <u>2 9/10 cm or 2.9 cm</u>

4.
E L

Measure = <u>1 6/10 = 1 3/5 cm or 1.6 cm</u>

Measure each angle in degrees, using a protractor, and record each measure.

5. Measure = <u>139°</u>

6. Measure = <u>22°</u>

7. Measure = <u>44°</u>

8. Define *congruent*: <u>Answers will vary, e.g., "Having the same size and shape."</u>
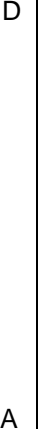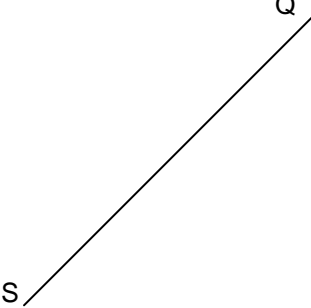
Name: _____

Determining Congruence
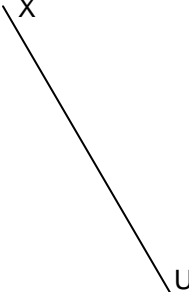
Part A

Measure each line segment in each pair, using a centimeter ruler, and record each measurement in the table. Then, determine whether the two line segments in each pair are congruent or noncongruent. Record your answers in the table.

1.

   V   J   D   A

2.

  Q   C   P   S

3.

  X   O   H   U

Name: _____

Determining Congruence

Part B

Measure each angle in each pair, using a protractor, and record each measurement in the table. Then, determine whether the two angles in each pair are congruent or noncongruent. Record your answers in the table.

4.
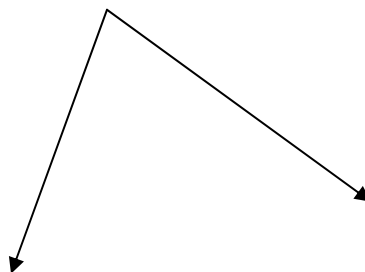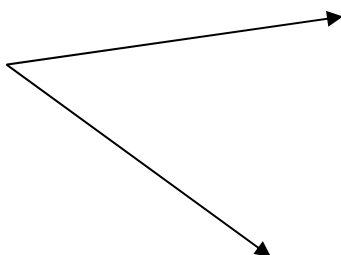
5.
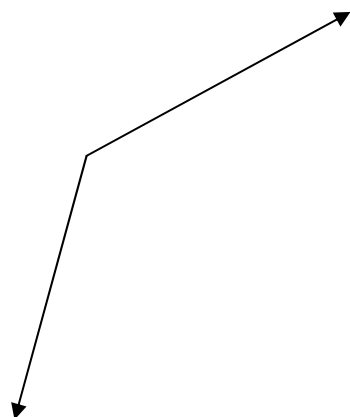
6.
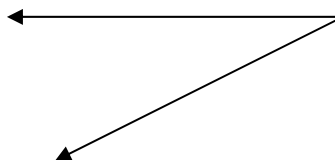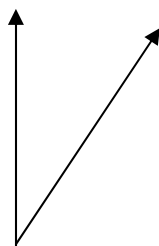
Name: _____

Determining Congruence

Part C

Trace one of the polygons in each pair, using patty paper or tracing paper and a marker. Then, place the tracing over the other polygon in the pair to determine whether the two polygons are congruent or noncongruent. Record your answers in the table.

7.
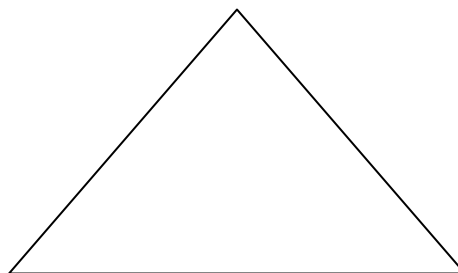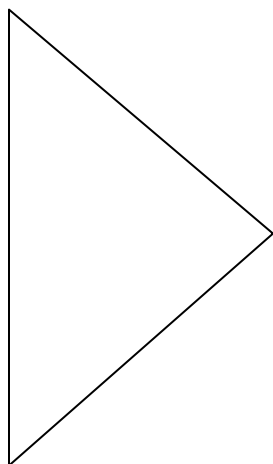
8.
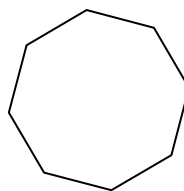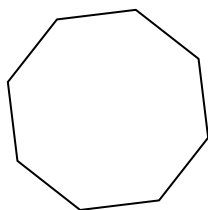
9.
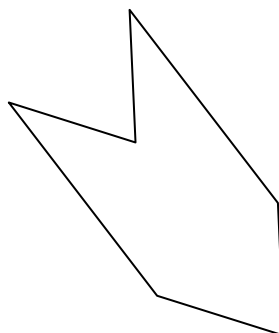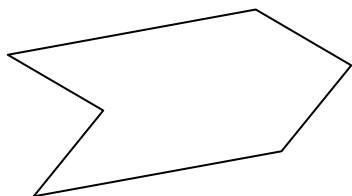
Name: _____

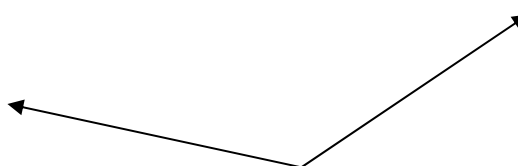# Determining Congruence Tables

Part A

Number	Measure Line Segment 1	Measure Line Segment 2	Congruent or Noncongruent
1			
2			
3			

Part B

Number	Measure Angle	Measure Angle 2	Congruent or Noncongruent
4			
5			
6			

Part C

Number	Congruent or Noncongruent
7	
8	
9	

Name: <u>**ANSWER KEY**</u>

Determining Congruence Tables

Part A

Number	Measure Line Segment 1	Measure Line Segment 2	Congruent or Noncongruent
1	<u>6 7/10 cm</u> <u>or</u> <u>6.7 cm</u>	<u>6 7/10 cm</u> <u>or</u> <u>6.7 cm</u>	<u>Congruent</u>
2	<u>5 4/10 = 5 2/5 cm</u> <u>or</u> <u>5.4 cm</u>	<u>5 7/10 cm</u> <u>or</u> <u>5.7 cm</u>	<u>Noncongruent</u>
3	<u>4 4/10 = 4 2/5 cm</u> <u>or</u> <u>4.4 cm</u>	<u>4 1/10 cm</u> <u>or</u> <u>4.1</u>	<u>Noncongruent</u>

Part B

Number	Measure Angle	Measure Angle 2	Congruent or Noncongruent
4	<u>45°</u>	<u>74°</u>	<u>Noncongruent</u>
5	<u>134°</u>	<u>134°</u>	<u>Congruent</u>
6	<u>33°</u>	<u>27°</u>	<u>Noncongruent</u>

Part C

Number	Congruent or Noncongruent
7	<u>Congruent</u>
8	<u>Noncongruent</u>
9	<u>Congruent</u>

Name: _____

Reflection

1. To the right is a practice SOL question. Circle your answer.

Which figures appear to be congruent?

2. Explain why you chose that answer.

    _____

    _____

    _____

    _____
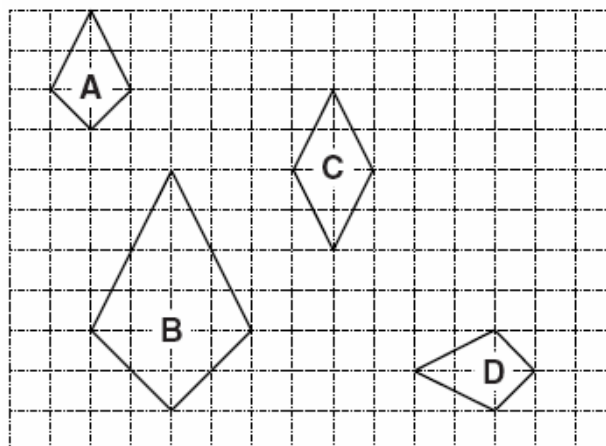
A   A and B
B   B and D
C   C and D
D   D and A

3. Draw two congruent line segments.

4. Draw two congruent angles.

Name: <u>**ANSWER KEY**</u>

Reflection

1. To the right is a practice SOL question. Circle your answer.

    <u>D</u>

    Which figures appear to be congruent?

2. Explain why you chose that answer.

    <u>Answers will vary, e.g. "A and D are the same size." "A and D are made up of the same number of grid boxes."</u>

    A   A and B
    B   B and D
    C   C and D
    Ⓓ   D and A

3. Draw two congruent line segments.

4. Draw two congruent angles.

✱ SOL 5.13, 5.14, 6.13b

Prerequisite SOL

6.13a

Lesson Summary

Students measure the angles inside a triangle and then classify the triangle as either acute, obtuse, or right. (60 minutes)

Materials

Protractors   "Triangle Classification" worksheets
"Warm-up" worksheets   "Triangle Classification Table" worksheets
"Triangle Vocabulary" handouts   "Reflection" worksheets

Vocabulary

polygon. A closed plane figure in which all sides are line segments.
triangle. A three-sided polygon.
acute triangle. A triangle that has three acute angles.
obtuse triangle. A triangle that has one obtuse angle.
right triangle. A triangle that has one right angle.

Warm-up

Distribute the "Warm-up" worksheets, and remind students that the sum of the measures of the three angles in a triangle equals 180°. You may wish to give them an example to help them remember this fact. Review how to find the measure of the third angle in a triangle when the other two angle measures are known. Have students complete the worksheet, and go over the answers before going on to the lesson.

Lesson

1. Distribute the "Triangle Vocabulary" handouts, and explain to or review with students that a triangle is classified by its largest angle. Go over the terms on the handout.

2. As a class, classify the four triangles on the "Warm-up" worksheet.

3. Explain and demonstrate how to extend the sides of a triangle to measure the angles and how to measure each angle inside the triangle to get all three angle measurements.

4. Distribute copies of the "Triangle Classification" and the "Triangle Classification Table" worksheets. The "Triangle Classification" worksheet is two pages and was designed to give the students space to extend the sides of the triangles for more accurate measuring.

5. Have students measure all three angles of each triangle and then classify each triangle. Have them record their answers on the "Triangle Classification Table" worksheet. Make sure students check to see that all three angles do add up to 180°.

Reflection

Have students complete the "Reflection" worksheet.

Name: _____

Warm-up

Find the measure of the missing angle in each triangle below. Do not use a protractor.
Remember: The measures of the three angles in a triangle add up to 180°.

1.

60°

90°   X

Measure of ∠X = _____

2.

B

56°   56°

Measure of ∠B = _____

3.

M

35°

119°

Measure of ∠M = _____

4.

K

47°   48°
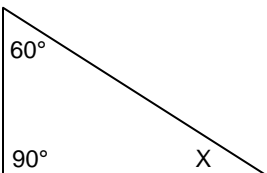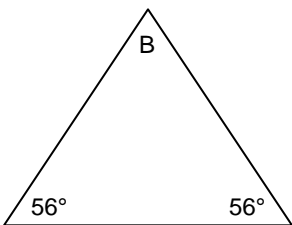
Measure of ∠K = _____

Name: <u>**ANSWER KEY**</u>
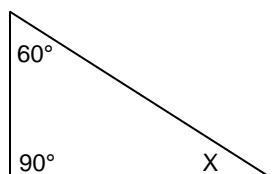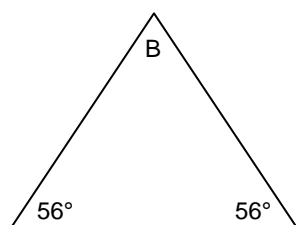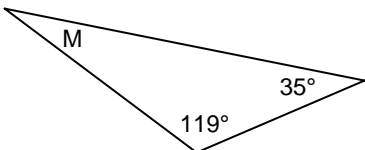
Warm-up
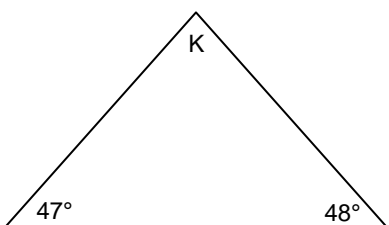
Find the measure of the missing angle in each triangle below. Do not use a protractor.
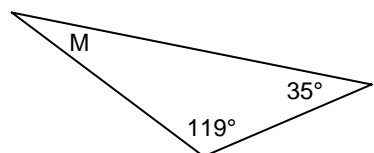***Remember: The measures of the three angles in a triangle add up to 180°.***
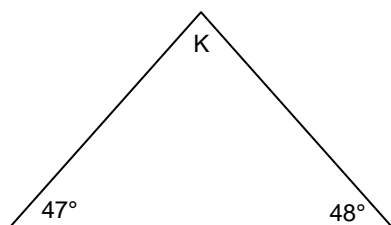
1.

60°

90°   X

Measure of ∠X = <u>30°</u>

2.

B

56°   56°

Measure of ∠B = <u>68°</u>

3.

M

35°

119°

Measure of ∠M = <u>26°</u>

4.

K

47°   48°

Measure of ∠K = <u>85°</u>

Triangle Vocabulary

polygon

A closed plane figure in which all the sides are line segments.

triangle

A three-sided polygon.
The sum of the three angles of a triangle equals 180°.

acute triangle

A triangle with three acute angles.

obtuse triangle

A triangle with one obtuse angle.

right triangle

A triangle with one right angle.

Remember: A triangle is classified by the *largest* of the three angles that form the triangle.

Name: _____

Triangle Classification

Measure each of the angles in each triangle using a protractor. Extend the sides if necessary. Make sure that the measures of the three angles add up to 180°. Record your answers in the Triangle Classification Table. Once you have figured out the measurement of each angle in the triangle, classify the triangle as acute, obtuse, or right.

1.
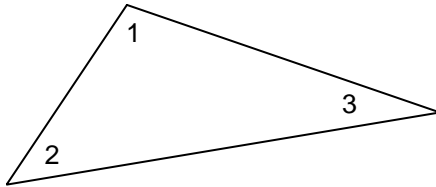
2.
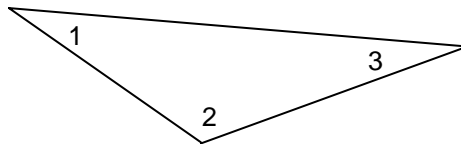
3.
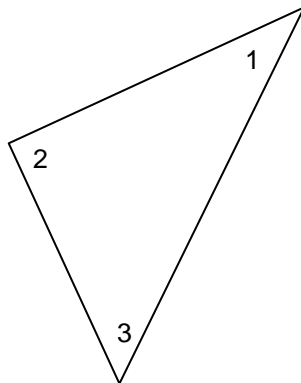
Name: _____

Triangle Classification

4.
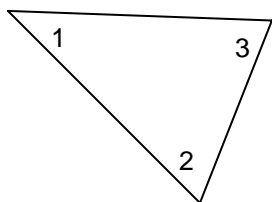
5.

6.

Name: _____

Triangle Classification Table

Triangle Number	Angle 1	Angle 2	Angle 3	Sum of the three angles	Classification
1					
2					
3					
4					
5					
6					

Name: <u>ANSWER KEY</u>

Triangle Classification Table

Triangle Number	Angle 1	Angle 2	Angle 3	Sum of the three angles	Classification
1	<u>103°</u>	<u>49°</u>	<u>28°</u>	<u>180°</u>	<u>obtuse</u>
2	<u>30°</u>	<u>125°</u>	<u>25°</u>	<u>180°</u>	<u>obtuse</u>
3	<u>45°</u>	<u>90°</u>	<u>45°</u>	<u>180°</u>	<u>right</u>
4	<u>45°</u>	<u>65°</u>	<u>70°</u>	<u>180°</u>	<u>acute</u>
5	<u>75°</u>	<u>85°</u>	<u>25°</u>	<u>180°</u>	<u>acute</u>
6	<u>37°</u>	<u>97°</u>	<u>46°</u>	<u>180°</u>	<u>obtuse</u>

Name: _____

Reflection

1. To the right are two practice SOL questions. Circle your answers.

    The largest angle in Δ *ABC* measures 104°. What kind of triangle is Δ *ABC*?

    A   Equiangular
    B   Obtuse
    C   Right
    D   Acute

2. Explain why you chose that answer to the first question.

    _____

    _____

    _____

    _____

    The angles in Δ *ABC* measure 27°, 73°, and 80°. What kind of triangle is Δ *ABC*?

    F   Equiangular
    G   Acute
    H   Obtuse
    J   Right

3. Explain why you chose that answer to the second question.

    _____

    _____

    _____

4. Can a triangle have more than one right angle? _____ Explain.

    _____

    _____

    _____

Name: <u>**ANSWER KEY**</u>

Reflection

1. To the right are two practice SOL questions. Circle your answers.

    <u>B, G</u>

2. Explain why you chose that answer to the first question.

    <u>A triangle is classified by its largest angle. A 104° angle is an obtuse angle. Therefore, the triangle is an obtuse triangle.</u>

3. Explain why you chose that answer to the second question.

    <u>All the angles in the triangle are acute. Therefore, the triangle is an acute triangle.</u>

The largest angle in Δ *ABC* measures 104°. What kind of triangle is Δ *ABC*?

A   Equiangular
(B) Obtuse
C   Right
D   Acute

The angles in Δ *ABC* measure 27°, 73°, and 80°. What kind of triangle is Δ *ABC*?

F   Equiangular
(G) Acute
H   Obtuse
J   Right

4. Can a triangle have more than one right angle?   <u>No</u>   Explain.

    <u>Two right angles add up to be 180°, and because the *three* angles of a triangle must add up to be 180°, there could not be another angle.</u>

✱ SOL 8.6

Prerequisite SOL

5.13, 5.14, 6.13

Lesson Summary

Students review complementary and supplementary angles and are introduced to vertical angles. (40 minutes)

Materials

"Warm-up" worksheets

"Angle Vocabulary" handouts

"Angle Relationships Formed by Two Intersecting Lines" worksheets

"Reflection" worksheets

Protractors

Colored pencils

Vocabulary

intersecting lines. Lines that cross each other and meet at a common point.

complementary angles. Two angles such that the sum of their measures is 90°.

supplementary angles. Two angles such that the sum of their measures is 180°.

vertical angles. Angles opposite one another at the intersection of two lines. Vertical angles are congruent.

Warm-up

Distribute the "Warm-up" worksheets, and if necessary, review the definitions of *complementary angles* and *supplementary angles*. Show students how to find a complement or supplement from the information given, and have them complete the worksheet. If they need extra practice in finding complementary and supplementary angles, use the additional-practice worksheet included. Go over the answers before going on to the lesson.

Lesson

1. Ask, "What does *intersect* mean?" List responses, and make sure that they clearly understand the term before moving on.

2. Distribute the "Angle Relationships Formed by Two Intersecting Lines" worksheet. Ask students, "How many angles are formed when two lines intersect?"

3. Have students use a protractor to measure each of the four numbered angles formed from the two intersecting lines. Have them record their information in the measurement box. Make sure that students realize that an angle may also be named by a number, if given. Also, students need to realize that the notation (m∠1) means "measure of angle 1."

4. Go over the answers. Ask students, "Which pairs of angles have the same measure?" (∠1 and ∠4, ∠2 and ∠3). Have students color ∠1 and ∠4 the same color and ∠2 and ∠3 a second color. Explain that the angle pairs that they have just colored are *vertical angles.* Vertical angles are always congruent, meaning that they have the same measure.

5. Distribute copies of the "Angle Vocabulary" handout. Have students fill in the notes box with this information.

6. Finish the notes box with the students, explaining the supplementary angle pairs. It may help students to cover ∠1 and ∠2 with their hands or another piece of paper to understand that ∠3 and ∠4 are supplementary, equal 180° when added together, and form a straight line.

Reflection

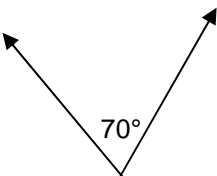Have students complete the "Reflection" worksheet.

Name: _____

Warm-up

Use the following definitions to find the complement and supplement of the given angles.

complementary angles. Two angles such that the sum of their measures is 90°.
supplementary angles. Two angles such that the sum of their measures is 180°.

1. 70°   Complement: _____   Supplement: _____

2. 37°   Complement: _____   Supplement: _____

3. 143°   Complement: _____   Supplement: _____

4. 160°   Complement: _____   Supplement: _____
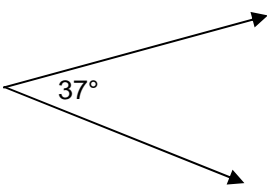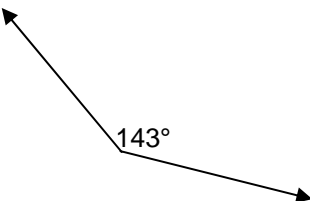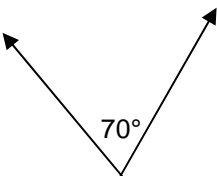
Name: <u>**ANSWER KEY**</u>
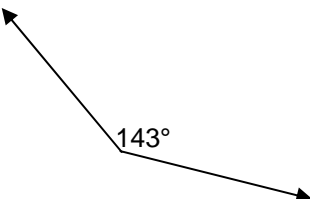
Warm-up

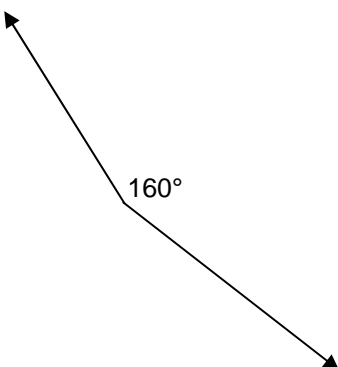Use the following definitions to find the complement and supplement of the given angles.

complementary angles. Two angles such that the sum of their measures is 90°.
supplementary angles. Two angles such that the sum of their measures is 180°.

1.   70° Complement: <u>20°</u> Supplement: <u>110°</u>

2.   37° Complement: <u>53°</u> Supplement: <u>143°</u>

3.   143° Complement: <u>none</u> Supplement: <u>37°</u>

4.   160° Complement: <u>none</u> Supplement: <u>20°</u>

Angle Vocabulary

complementary angles

Two angles such that the sum of their measures is 90°.

supplementary angles

Two angles such that the sum of their measures is 180°.
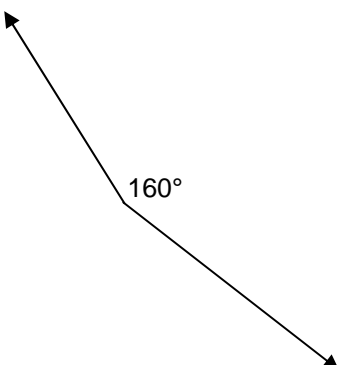
vertical angles

Angles opposite one another at the intersection of two lines. Vertical angles are congruent.

intersecting lines

Lines that cross each other and meet at a common point.

Name: _____

Angle Relationships Formed by
Two Intersecting Lines

m∠1 = _____

m∠2 = _____

m∠3 = _____

m∠4 = _____

∠1 and ∠4 are _____ angles.

∠2 and ∠3 are _____ angles.

∠1 and ∠3 are _____ angles.

∠1 and ∠2 are _____ angles.

∠2 and ∠4 are _____ angles.

∠3 and ∠4 are _____ angles.
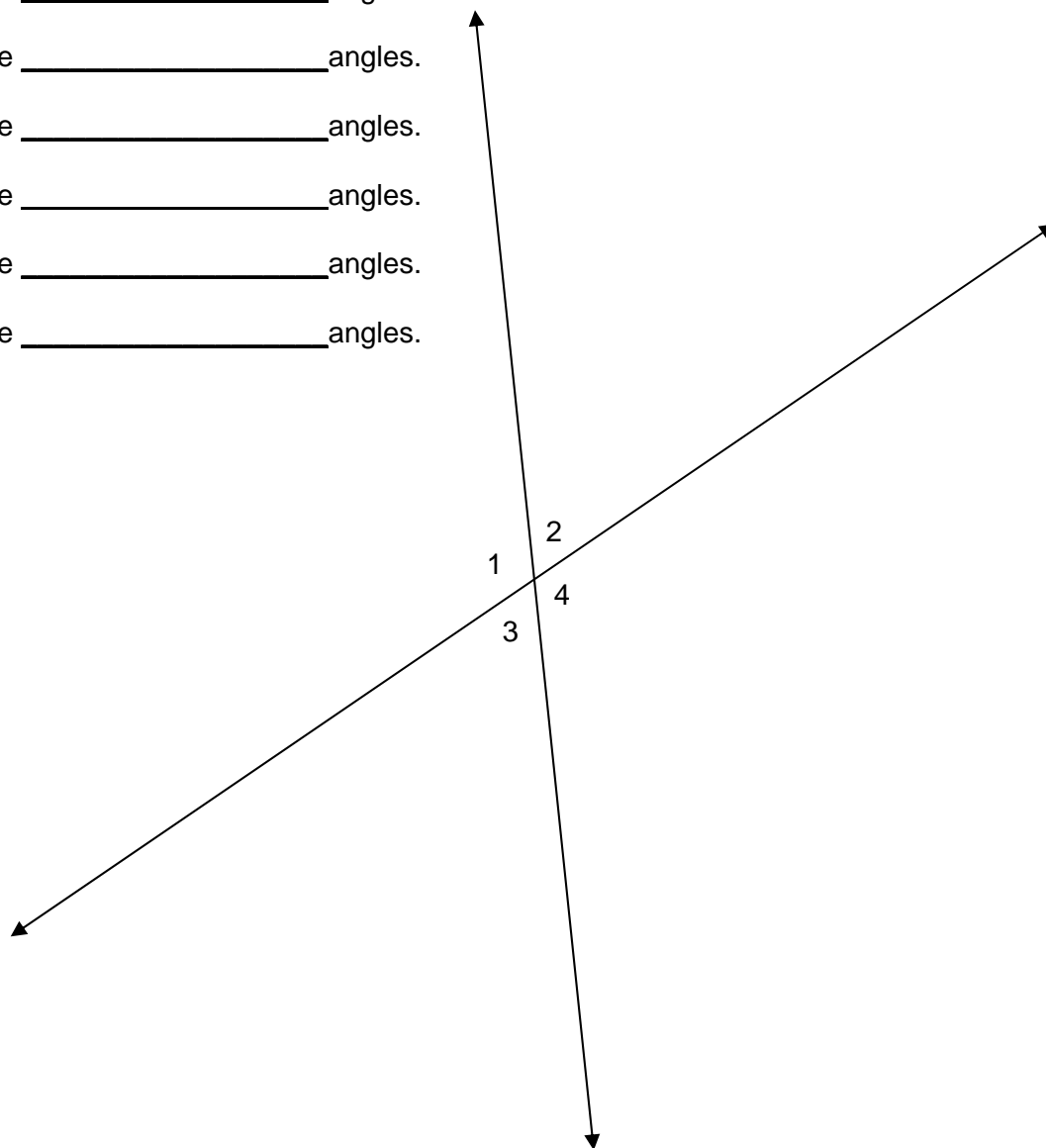
Name: <u>ANSWER KEY</u>

Angle Relationships Formed by Two Intersecting Lines

m∠1 = <u>118°</u>

m∠2 = <u>62°</u>

m∠3 = <u>62°</u>

m∠4 = <u>118°</u>

∠1 and ∠4 are <u>vertical (congruent)</u> angles.

∠2 and ∠3 are <u>vertical (congruent)</u> angles.

∠1 and ∠3 are <u>supplementary</u> angles.

∠1 and ∠2 are <u>supplementary</u> angles.

∠2 and ∠4 are s<u>upplementary</u> angles.

∠3 and ∠4 are <u>supplementary</u> angles.
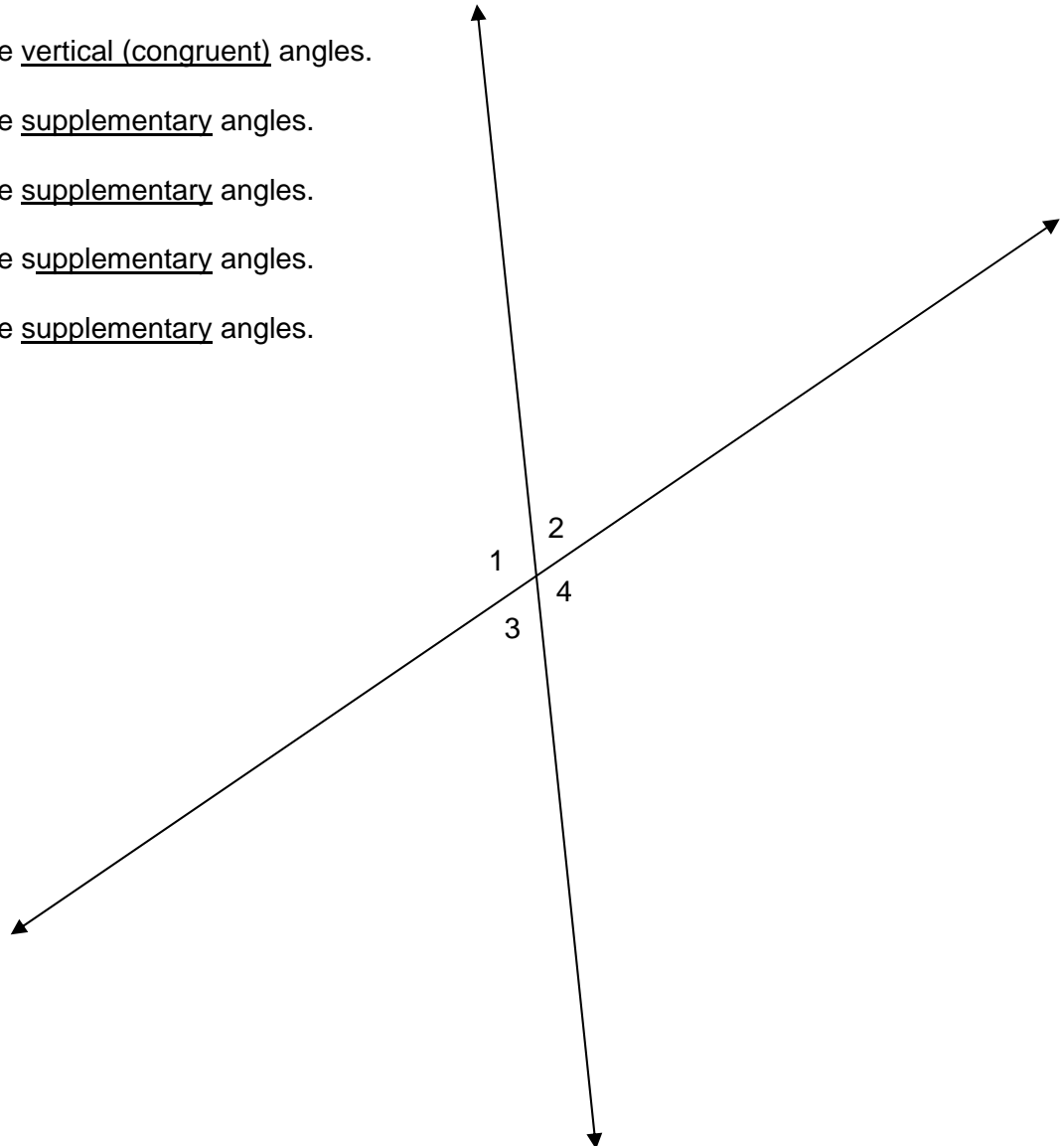
Name: _____

Additional Practice
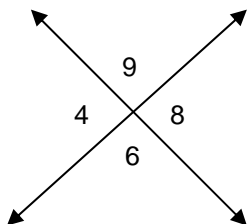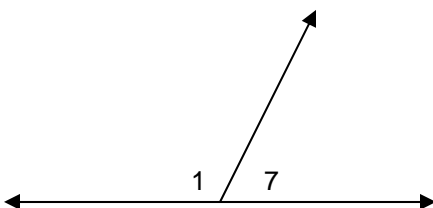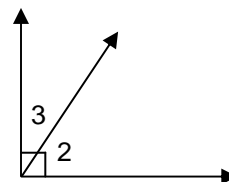
Figure 1

Figure 2

Figure 3

Part A: Use Figure 1 to answer the following questions:

1.   ∠9 and ∠6 are _____ angles.

2.   ∠4 and ∠6 are _____ angles.

3.   If m∠4 = 87°, then m∠8 = _____.

4.   If m∠4 = 87°, then m∠9 = _____.

Part B: Use Figure 2 to answer the following questions:

5.   ∠1 and ∠7 are _____ angles.

6.   m∠1 + m∠7 = _____.

7.   If m∠1 = 115°, then m∠7 = _____.

Part C: Use Figure 3 to answer the following questions:

8.   ∠2 and ∠3 are _____ angles.

9.   m∠2 + m∠3 = _____.

10.   If m∠3 = 40°, then m∠2 = _____.

Name: <u>**ANSWER KEY**</u>

Additional Practice
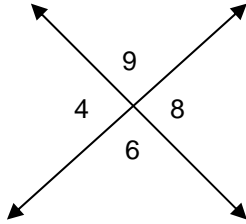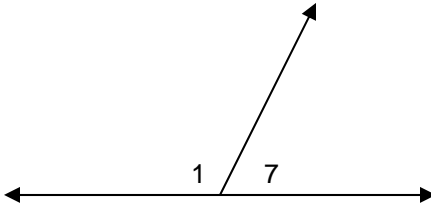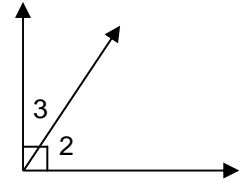
Figure 1   Figure 2   Figure 3

Part A: Use Figure 1 to answer the following questions:

1.   ∠9 and ∠6 are <u>vertical (congruent)</u> angles.

2.   ∠4 and ∠6 are <u>supplementary</u> angles.

3.   If m∠4 = 87°, then m∠8 = <u>87°</u>.

4.   If m∠4 = 87°, then m∠9 = <u>93°</u>.

Part B: Use Figure 2 to answer the following questions:

5.   ∠1 and ∠7 are <u>supplementary</u> angles.

6.   m∠1 + m∠7 = <u>180°</u>.

7.   If m∠1 = 115°, then m∠7 = <u>65°</u>.

Part C: Use Figure 3 to answer the following questions:

8.   ∠2 and ∠3 are <u>complementary</u> angles.

9.   m∠2 + m∠3 = <u>90°</u>.

10. If m∠3 = 40°, then m∠2 = <u>50°</u>.

Name: _____

Reflection

1. To the right is a practice SOL question. Circle your answer.

    In the diagram below, lines *l* and *p* intersect.
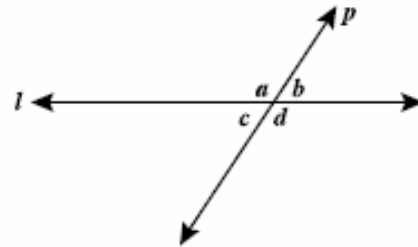
2. Explain why you chose that answer.

    _____
    _____
    _____
    _____

    If the measure of ∠*a* is 109°, what is the measure of ∠*b*?

3. Find the m∠d. Explain how you came up with your answer.

    _____
    _____
    _____
    _____

    F   109°

    G   100°

    H   71°

    J   19°

4. Find the m∠c. Explain how you came up with your answer.

    _____
    _____
    _____
    _____

Name: <u>ANSWER KEY</u>

Reflection

1. To the right is a practice SOL question. Circle your answer.

    <u>H</u>

2. Explain why you chose that answer.

    <u>∠b is supplementary to ∠a because together they form a straight line. This means that together they are 180°. 109° subtracted from 180° is 71°.</u>

3. Find the m∠d. Explain how you came up with your answer.

    <u>109°</u>
    <u>∠d is vertical to ∠a. Vertical angles are congruent. If ∠a is 109°, then so is ∠d.</u>

4. Find the m∠c. Explain how you came up with your answer.

    <u>71°</u>
    <u>∠c is vertical to ∠b. Vertical angles are congruent. If ∠b is 71°, then so is ∠c.</u>

In the diagram below, lines *l* and *p* intersect.
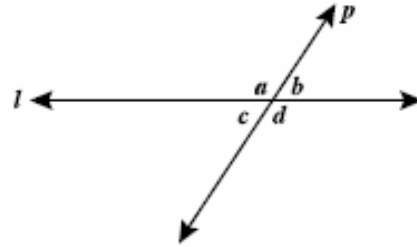
If the measure of ∠*a* is 109°, what is the measure of ∠*b*?

F   109°

G   100°

(H) 71°

J   19°

✳ SOL 6.16

Prerequisite SOL

5.13, 5.14, 6.13b, 6.15

Lesson Summary

Students use a ruler and compass to construct perpendicular lines and angle bisectors. Students check the accuracy of their constructions through direct measurement. (45 minutes) (Teacher Note: SOL 6.16 is not a tested standard and, therefore, will not be included in an ARDT strengths and weakness report.)

Materials

Centimeter rulers   "Warm-up" worksheets   "Reflection" worksheets
Compasses   "Bisector Notes and Practice" worksheets
Protractors   "Bisector Independent Practice" worksheets

Vocabulary

bisect. To divide into two equal parts.
bisector. A line that divides a figure into two congruent parts.
congruent. The same size and shape.
perpendicular lines. Two lines that intersect to form right angles.

Warm-up

Distribute the "Warm-up" worksheets, rulers, and protractors, and have students complete the worksheet. Review the answers, and ask for questions. Ask the students what they notice about ray PH and ray AW. (Ray PH divides ∠SPK into two angles that are congruent; ray AW divides line segment BJ in half and creates two 90° angles.) Review the definition of *bisector* as a line that divides a figure into two congruent parts, and explain to students that each of these rays is a bisector.

Lesson

1. Give each student a copy of the "Bisector Notes and Practice" worksheets and a compass. You may want to display a copy on the overhead so that students can watch you complete the steps. (Teacher Note: It is advisable to review rules for safe use of a compass.)

2. Go through the constructions step by step as you model a construction for the class. See the attached figures and step by step directions. Have the students do each step after you model it. (Teacher Note: While measurements in the examples are meant to be accurate, they may be slightly off due to printing anomalies. You may want to verify the measurements after printing the worksheets.)

3. Have students complete the "Bisector Independent Practice" on their own, and provide help as needed.

Reflection

Have students complete the "Reflection" worksheet.
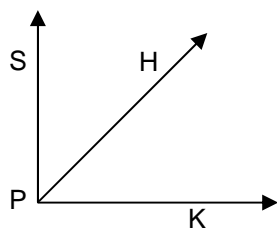
Name: _____

Warm-up

1.   m∠SPK = _____

2.   m∠SPH = _____

3.   m∠HPK = _____
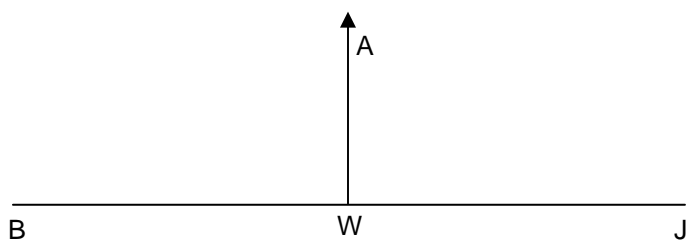
4.   The measure of line segment BJ in centimeters = _____

5.   The measure of line segment BW in centimeters = _____

6.   The measure of line segment WJ in centimeters = _____

7.   m∠BWA = _____

8.   m∠JWA = _____

Name: <u>ANSWER KEY</u>

Warm-up
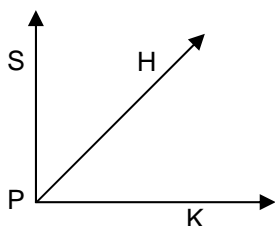
1. m∠SPK = <u>90°</u>

2. m∠SPH = <u>45°</u>

3. m∠HPK = <u>45°</u>

4. The measure of line segment BJ in centimeters = <u>10 cm</u>

5. The measure of line segment BW in centimeters = <u>5 cm</u>

6. The measure of line segment WJ in centimeters = <u>5 cm</u>

7. m∠BWA = <u>90°</u>

8. m∠JWA = <u>90°</u>

Name: _____

Bisector Notes and Practice

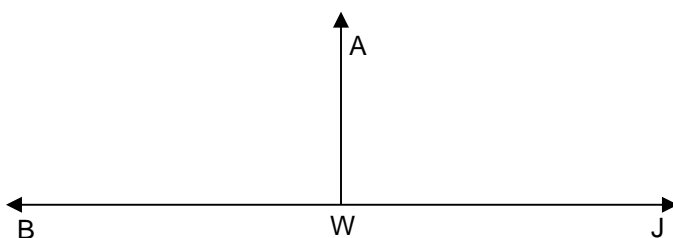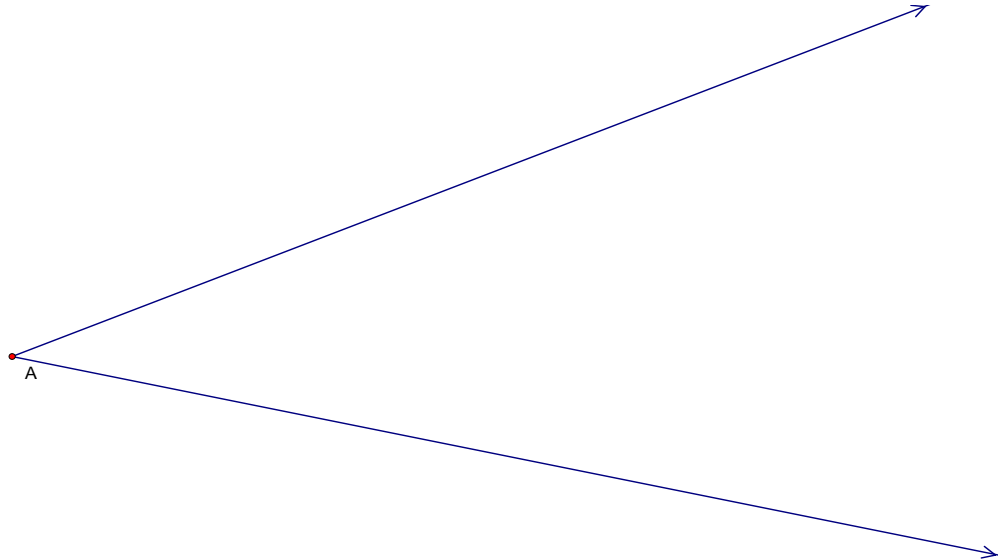1. Using a compass and a ruler, construct the angle bisector of the angle below by following the steps listed.

    A

    a. Open your compass to any position. Place the point of your compass on the vertex (point A), and draw a complete circle. This circle should intersect the angle at two places. Label those points B and C.
    b. Make sure that the opening of your compass has not changed. Place the point of your compass on point B, and draw a full circle.
    c. Make sure that the opening of your compass has not changed. Place the point of your compass on point C, and draw a full circle. The two circles created in steps b and c should intersect at a point. Label that point D.
    d. Use your ruler to create a ray from point A through point D and beyond. This ray is the *angle bisector*.

2. m∠BAC = _____

3. m∠BAD = _____

4. m∠DAC = _____

Name: _____

Bisector Notes and Practice

5.   Using a compass and a ruler, construct the perpendicular bisector to the line segment below by following the steps listed.

B   A

a.   Place your compass point on point B and open the compass until the pencil point touches point A. This will create a radius the size of the line segment.
b.   With your compass point still on point B, draw a whole circle.
c.   Make sure the opening of your compass has not moved. Place your compass point on point A, and draw a whole circle. There should be two points where the two circles intersect. Label those points C and D.
d.   Connect points C and D, creating line segment CD. This line segment is the *perpendicular bisector* of line segment BA.

6.   Line segment BA in centimeters = _____

7.   Line segment BC in centimeters = _____

8.   Line segment CA in centimeters = _____

Name: <u>ANSWER KEY</u>

Bisector Notes and Practice

1. Using a compass and a ruler, construct the angle bisector of the angle below by following the steps listed.

   a. Open your compass to any position. Place the point of your compass on the vertex (point A), and draw a complete circle. This circle should intersect the angle at two places. Label those points B and C.
   b. Make sure that the opening of your compass has not changed. Place the point of your compass on point B, and draw a full circle.
   c. Make sure that the opening of your compass has not changed. Place the point of your compass on point C, and draw a full circle. The two circles created in steps b and c should intersect at a point. Label that point D.
   d. Use your ruler to create a ray from point A through point D and beyond. This ray is the *angle bisector*.

2. m∠BAC = <u>35°</u>

3. m∠BAD = <u>17°</u>

4. m∠DAC = <u>17°</u>

Name: <u>ANSWER KEY</u>

Bisector Notes and Practice

5.   Using a compass and a ruler, construct the perpendicular bisector to the line segment below by following the steps listed.
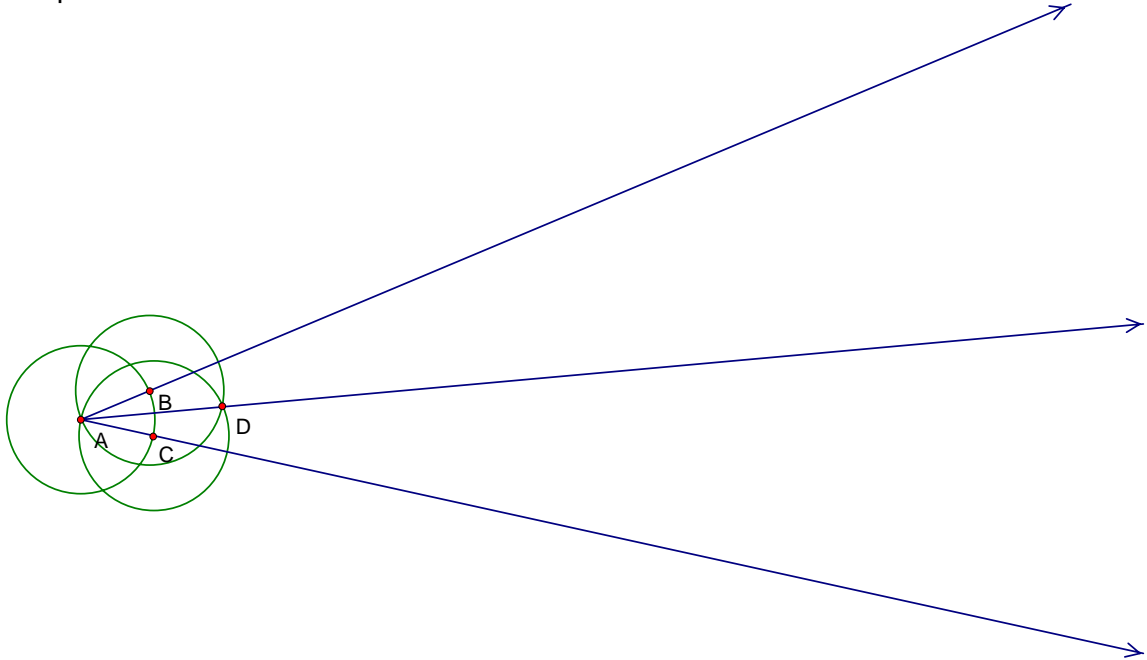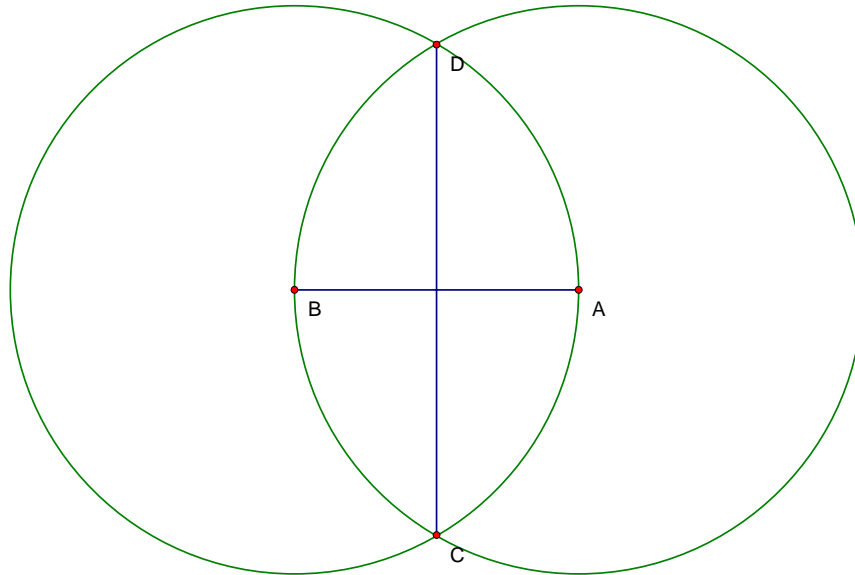

   a.   Place your compass point on point B and open the compass until the pencil point touches point A. This will create a radius the size of the line segment.
   b.   With your compass point still on point B, draw a whole circle.
   c.   Make sure the opening of your compass has not moved. Place your compass point on point A, and draw a whole circle. There should be two points where the two circles intersect. Label those points C and D.
   d.   Connect points C and D, creating line segment CD. This line segment is the *perpendicular bisector* of line segment BA.

6.   Line segment BA in centimeters = <u>4 2/10 or 4 1/5 cm</u>

7.   Line segment BC in centimeters = <u>2 1/10 cm</u>

8.   Line segment CA in centimeters = <u>2 1/10 cm</u>

Name: _____

Independent Practice on Constructing Bisectors

1. Use the steps on the "Bisector Notes and Practice" worksheet to construct the angle bisector of the angle below.

A
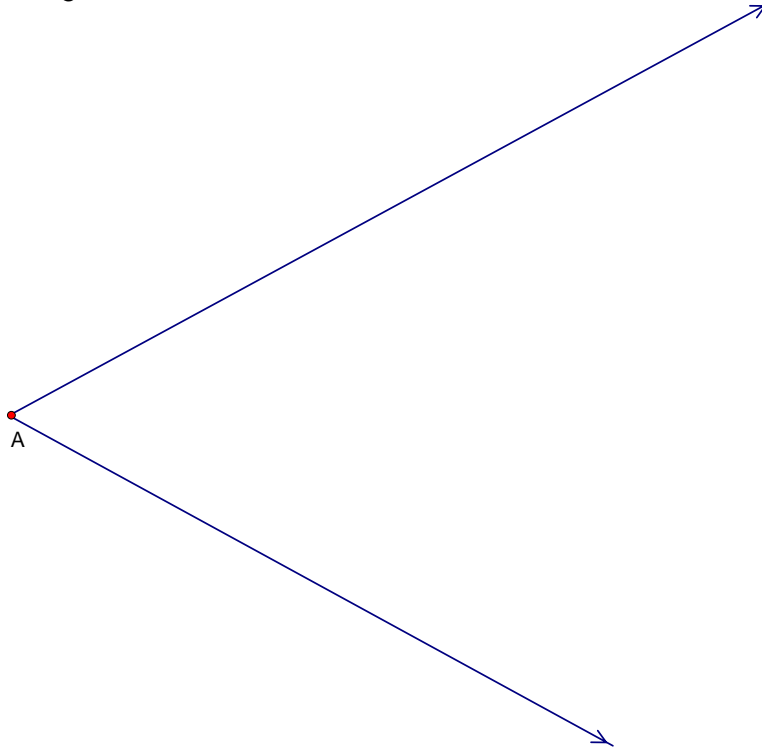
2. m∠BAC = _____

3. m∠BAD = _____

4. m∠DAC = _____

5. Use the steps on the "Bisector Notes and Practice" worksheet to bisect the line segment at right.

6. Measure of line segment BA in centimeters = _____

B   A

7. Measure of line segment BC in centimeters = _____

8. Measure of line segment CA in centimeters = _____

Name: <u>ANSWER KEY</u>

Independent Practice on Constructing Bisectors

1. Use the steps on the "Bisector Notes and Practice" worksheet to construct the angle bisector of the angle below.

2. m∠BAC = <u>57°</u>
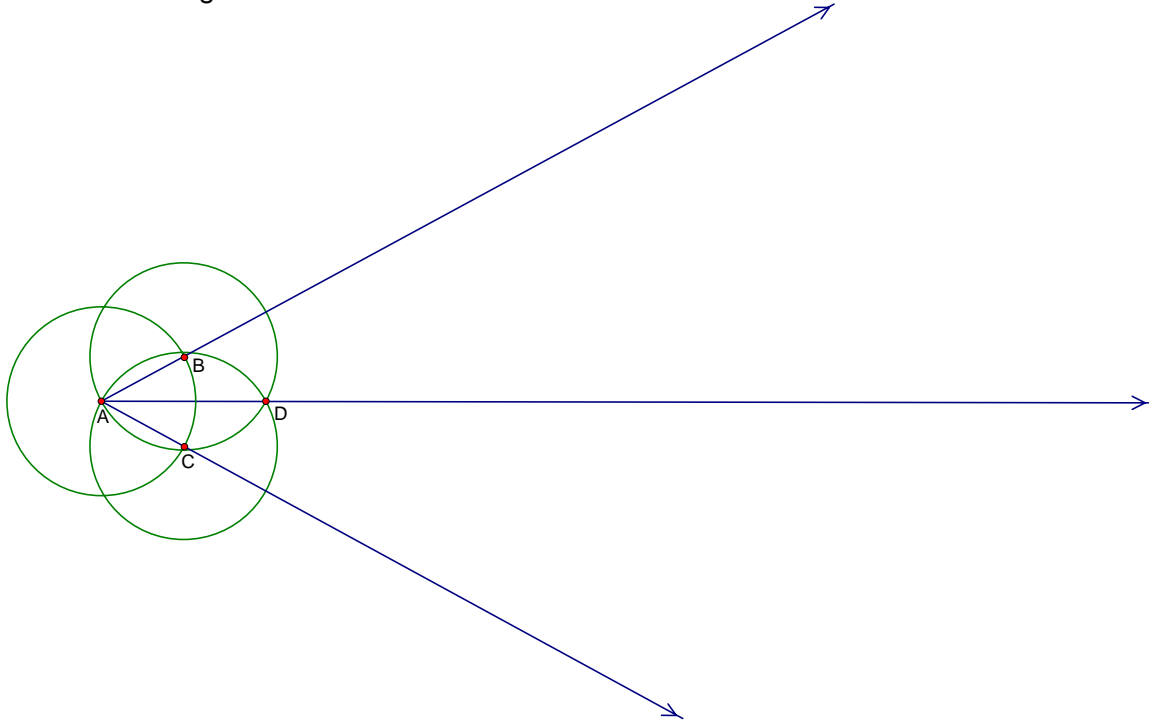
3. m∠BAD = <u>28°</u>

4. m∠DAC = <u>28°</u>

5. Use the steps on the "Bisector Notes and Practice" worksheet to bisect the line segment at right.

6. Measure of line segment BA in centimeters = <u>3 3/10 cm</u>

7. Measure of line segment BC in centimeters = <u>1 7/10 cm</u>

8. Measure of line segment CA in centimeters = <u>1 7/10 cm</u>

Name: _____

Reflection

1. Draw a line segment of your own, and then construct the perpendicular bisector of that line segment. Measure the original line segment and the two new line segments that are created by the perpendicular bisector.

2. Draw an angle of your own, and then construct the angle bisector of that angle. Measure the original angle and the two new angles that are created by the angle bisector.